TRINITY COLLEGE

Year 12 Mathematics Applications
Test 3 2020

Section 1 Calculator Free

Graphs and Networks

STUDENT'S NAME Solutions - Hantzis

DATE: Thursday 14th May **TIME:** 30 minutes **MARKS**: 27

INSTRUCTIONS:

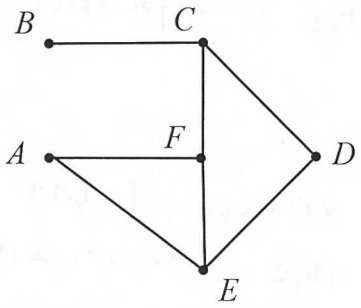
Standard Items: Pens, pencils, drawing templates, eraser

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

---

1. (6 marks)

A simple graph is shown below.



(a) Explain why the graph is simple. [2]

- No loops ✓
- No Multiple edges ✓

(b) Show that the graph satisfies Euler's formula [2]

$e = 7$ $f = 3$

$v = 6$ ✓

$6 + 3 - 7 = 2$

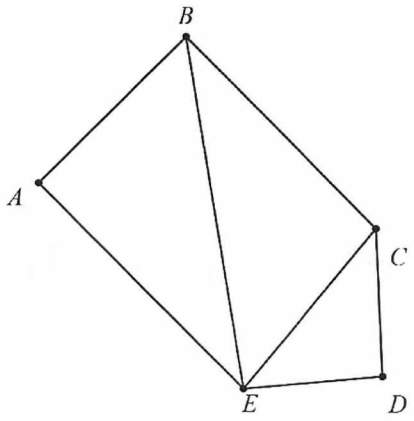
$2 = 2$ ✓

(c) Clearly explain why the graph contains a Hamiltonian path but not a Hamiltonian cycle. [2]

- Open path that visits every vertex once only ✓
  e.g. BCDEFA. (Defines Hamiltonian path).
- Because BC is a bridge, a cycle cannot exist. ✓
  (Identifies bridge).

2. (3 marks)

A graph below shows a network of homes A, B, C, D and E and the edges represent the drainpipes connecting their homes.



(a) What is the degree of C? [1]

3 ✓

(b) Give one explanation why:

(i) $ABEDCEBC$ is not a trail, [1]

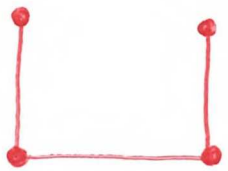
Edge BE is repeated ✓

(ii) The graph is semi-Eulerian, [1]

- Only 2 vertices of odd degree (B and C). ✓ (One valid reason of those provided)
- Every edge is traversed once
- Start at one odd vertex and finish at the other.

3. (4 marks)

(a) Sketch a connected graph which has two vertices of degree 1 and two vertices of degree 2. [2]



✓ 2 vertices of degree 1

✓ 2 vertices of degree 2.

(b) Is every edge in your graph in part (a) a bridge? Explain. [2]

Yes ✓

Every edge is connected, that if removed leaves the graph disconnected. ✓ (valid reason)

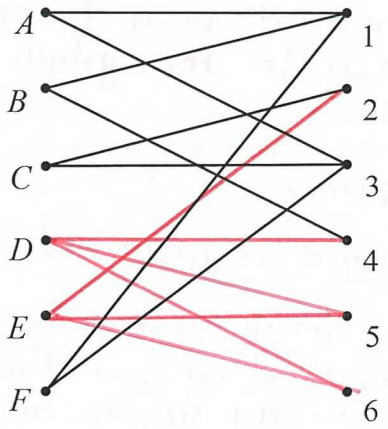
4. (5 marks)

Six people, Andy, Bob, Carl, Dan, Eric, and Fred are to be allocated to six tasks, 1, 2, 3, 4, 5 and 6.

The following table shows the tasks that each person can undertake.

Person	Task
Andy	1, 3
Bob	1, 4
Carl	2, 3
Dan	4, 5, 6
Eric	2, 5, 6
Fred	1, 3

(a) This information has been represented on a bipartite graph.
Complete the missing entries for Dan and Eric. [2]



√√ All correct

√ 1 error

(b) Each person is to be allocated to only one task and all tasks need to be completed.

(i) Initially Bob is allocated to task 1, Carl to task 3, Dan to task 5 and Eric to task 2.

Explain the problem with this allocation. [1]

Tasks 4 and 6 cannot be completed as Bob, Dan and Eric have been allocated to other task.

√ identifies Task 4 and/or 6 cannot be completed as members are allocated elsewher [2]

(ii) State *an* allocation where each person is given only one task.

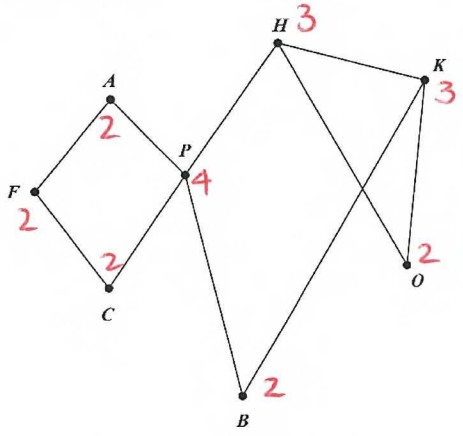
Andy – 1
Bob – 4
Carl – 2
Dan – 5
Eric – 6
Fred – 3

√√ All correct

√ 1-2 errors

5. (9 marks)

The graph below shows the network of flights provided by Qantas Link on a given week to the Regional mines of Western Australia.



(a) Determine with reasons if the edge PH is a bridge. [2]

No, PH is not a bridge. ✓

If PH is removed both P and H are still connected to the graph. ✓

✓ States PH is not a bridg
✓ Defines a bridge

(b) Determine with reasons if this graph contains a semi-Eulerian trail. [2]

HPAFCPBKHOK

Yes the graph contains a semi-Eulerian trail.

Every edge can be traversed using every edge exactly once, starting and finishing at different vertices of odd degree being H and K.

✓ Every edge traversed only once
✓ start and finish at different vertices of odd degree H and K

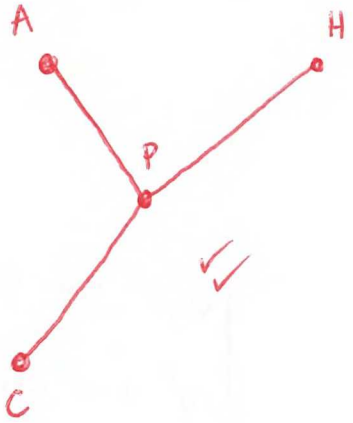
(c) Determine with reasons which edge you would remove so that this graph forms an Eulerian circuit. Give a possible Eulerian circuit. [3]

HK → All vertices would be even making the Graph Eulerian, and a circuit as all edges are traversed once and start and finish at the same vertex

Possible Circuit: FCPBKOHPAF.

✓ HK edge
✓ Defines Eulerian circuit
✓ Gives a circuit

(d) In the space provided below, draw a graph containing the vertices A,P,C and H with 3 edges, all of which are bridges. [2]



✓ All vertices Listed
✓ All edges are bridges

TRINITY COLLEGE

Year 12 Mathematics Applications
Test 3 2020

Section 2 Calculator Assumed

Graphs and Networks

STUDENT'S NAME _____

DATE: Thursday 14th May **TIME:** 20 minutes **MARKS**: 21

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

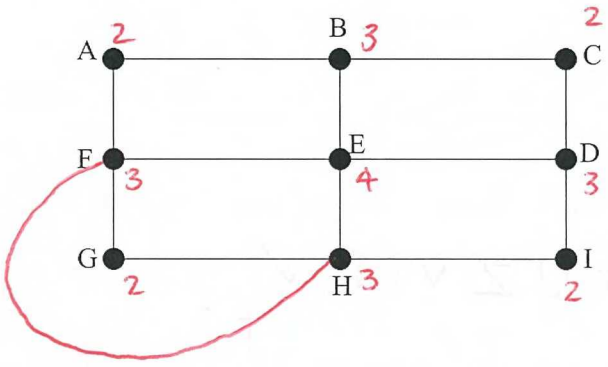
Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

---

6. (4 marks)

The local council wants the road works manager to inspect each of the new roads in a section of a new development as shown in the diagram below.



(a) Why is it not possible for the manager to inspect all the roads exactly once? Explain. [2]

- Graph does not contain an Eulerian trail or circuit.
- More than two vertices of odd degree.

[✓ Identifies the graph does not contain Eulerian trail or circuit. ✓ Identifies more than two vertices of odd degree]

(b) A road is to built between two vertices to allow all roads to be inspected exactly once. Where should it be built? Indicate clearly on the original diagram above.. [1]

On Graph [✓ connects any two vertices of odd degree]

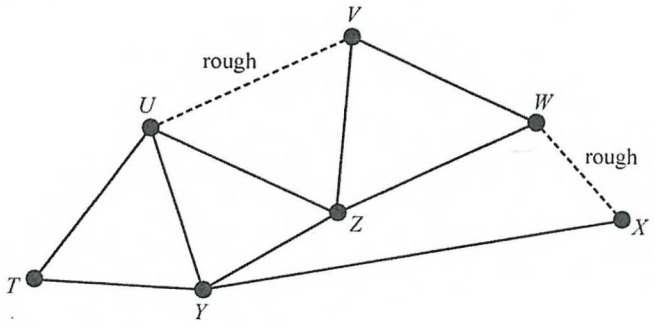
(c) What is the minimum number of roads that need to be added to the original graph so that the inspection can start and end at the same location? [1]

2 ✓ [✓ 2 roads to make all vertices of even degree]

7. (5 marks)

The suburb of Kingsford has a skateboard park with seven ramps.

The ramps are shown as vertices T, U, V, W, X, Y and Z on the graph below.



The tracks between ramps U and V and between ramps W and X are rough, as shown on the graph above.

(a) George begins skating at ramp W and follows an Eulerian trail. He can skate over any of the tracks.

At which ramp does George finish? [1]

✓ ✓

[✓ identifies vertex ✓]

(b) Frank begins skating at ramp X and follows a Hamiltonian path. The path he chooses does not include the two rough tracks.

Write down a path that Frank could take from start to finish. [2]

X Y T U Z V W ✓

[or X Y T U Z V W]

(c) Brett wants to complete a Hamiltonian cycle starting from X without traversing the rough paths. Is this possible? Explain. [2]

No, this is not possible as in order to complete a Hamiltonian cycle Brett would need to start and end at the same vertex X and in order to complete this he would need to traverse the rough path.

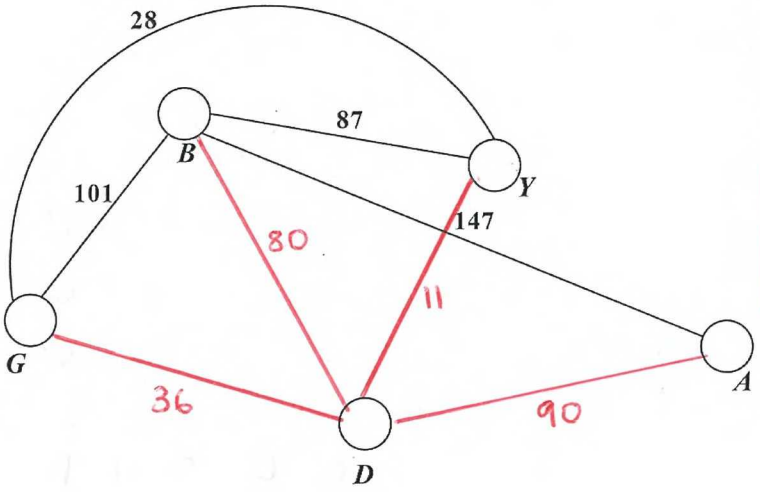
[✓ identifies it is not possible.
✓ Articulates the need to start and end at the same vertex would require rough path]

8. (7 marks)

The table below shows the distances (km) between the towns to be visited as a part of a Geography field trip.

	Bunbury (B)	Yallingup (Y)	Augusta (A)	Dunsborough (D)	Gracetown (G)
Bunbury	-	87	147	80	101
Yallingup	87	-	-	11	28
Augusta	147	-	-	90	-
Dunsborough	80	11	90	-	36
Gracetown	101	28	-	36	-

(a) Using the data shown in the table, complete the weighted graph below for vertex D. [3]



√√√ All edges and weights correct
√√ One error
√ Two errors

(b) A Tourist would like to visit all towns once, without travelling any road more than once. They are happy to start and finish at different towns.

(i) Identify the shortest path and state its length. [3]

A - D - Y - G - B

= 230 km

√√ Path includes all Towns
√ 230km

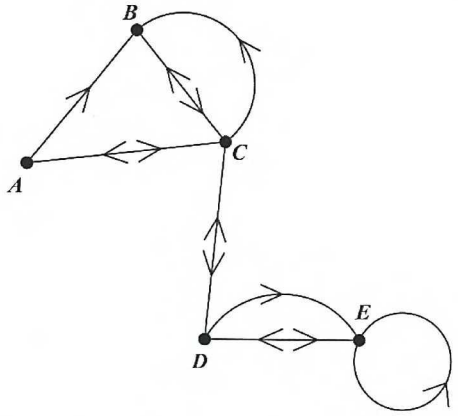
(ii) What name is given to the route in part (b)? [1]

Semi- Hamiltonian Path ✓

(Open - Hamiltonian Path)

9. (5 marks)

The diagram below shows the graph of the roads surrounding the Education buildings of the University of Notre Dame.



(a) Complete the adjacency matrix M below for vertex E. [2]

$M =$

From \ To	A	B	C	D	E
A	0	1	1	0	0
B	0	0	1	0	0
C	1	2	0	1	0
D	0	0	1	0	2
E	0	0	0	1	1

[✓✓ All correct
✓ One error]

(b) What feature of the matrix indicates it is a directed graph? [1]

Entries not symmetrical across the leading diagonal ✓

[✓ Articulates not symmetrical]

(c) Using the matrix M^2

$M^2 =$

From \ To	A	B	C	D	E
A	1	2	1	1	0
B	1	2	0	1	0
C	0	1	4	0	2
D	1	2	0	3	2
E	0	0	1	1	3

(i) State the connection which has the greatest number of walks of length 2. [1]

C to C ✓

(ii) The number of walks of length 2 between the vertices D and E starting at either vertex. [1]

3 ✓